What I know about lines? **A Review of Chapter 5: Linear Equations**

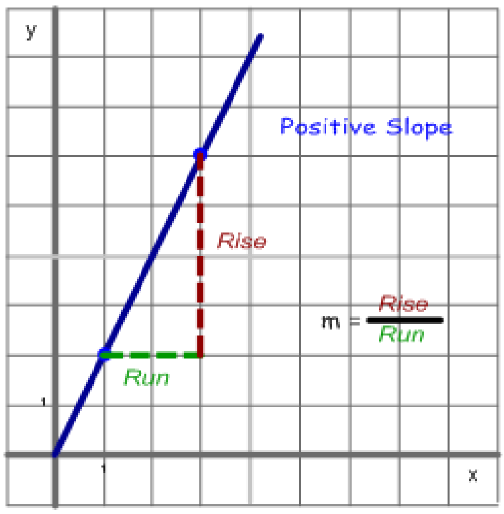
A line can be represented graphically on a coordinate plane or in an algebraic equation.

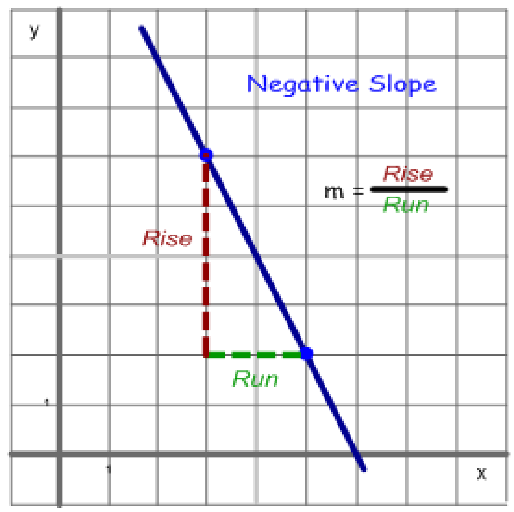
A line consists of at least two points or coordinates.

A coordinate [or point] is a location on the coordinate plane. It is composed of an x-coordinate and a y-coordinate, (x, y) .

Label individual points with subscripts, examples:

Point 1 (x_1, y_1) Point 2 (x_2, y_2) Point A (x_A, y_A) Point B (x_B, y_B)





Find the equation of the line between two points using the graphing method.

1. Plot any two points on a coordinate grid. Label your axis and points.

2. Draw a line connecting the two points.

3. Create a Right Triangle between the two points (draw a horizontal & vertical line at a right angle)

4. Count the distance between the right angle and each point. Label the Rise and Run with values.

5. Find the slope, m. $m = \frac{Rise}{Run}$ Or compute it with $m = \frac{y_2 - y_1}{x_2 - x_1}$.

6. Determine if the line is positive slope or negative slope. Read the graph from right to left.

7. Find where the line crosses the y-axis. This is the y-intercept $(0, b)$.

8. Now write the equation $y = mx + b$ *Slope – Intercept Form*

All the Forms of the Equation of the Line:

$$y = \boldsymbol{m}x + \boldsymbol{b} \qquad \textbf{\textit{Slope} – \textit{Intercept Form}}$$

$$\boldsymbol{A}x + \boldsymbol{B}y = \boldsymbol{C} \qquad \textbf{\textit{Standard Form}}$$

$$y - \boldsymbol{y_1} = \boldsymbol{m}(x - \boldsymbol{x_1}) \qquad \textbf{\textit{Point} – \textit{Slope Form}}$$

The **y-intercept** is the point [or coordinate] $(\mathbf{0}, \boldsymbol{y})$ or $(\mathbf{0}, \boldsymbol{b})$ where a line crosses the y-axis. At this point the x-coordinate is zero and the y-coordinate is b and the unknown value.

The **x-intercept** is the point [or coordinate] $(\boldsymbol{x}, \mathbf{0})$ where a line crosses the x-axis. At this point the y-coordinate is zero and the x-coordinate is the unknown value.

Horizontal lines cross only the y-axis. Therefore the equation of the line is y = b, the y-intercept for a horizontal line is $(\mathbf{0}, \boldsymbol{b})$.

There ***isn't any rise (No Rise)*** in a horizontal line therefore

the slope is $m = \frac{Rise}{Run} = \frac{\mathbf{0}}{run} = \mathbf{0}$, zero.

Advance Concepts

$$y = \mathbf{0} \cdot x + \boldsymbol{b} \qquad \textbf{\textit{Slope} – \textit{Intercept Form}} \qquad y = \boldsymbol{b}$$

$$\mathbf{0} \cdot x + \boldsymbol{B}y = \boldsymbol{C} \qquad \textbf{\textit{Standard Form}} \qquad y = \frac{C}{B} = \boldsymbol{b}$$

$$y - \boldsymbol{y_1} = \mathbf{0} \cdot (x - \boldsymbol{x_1}) \qquad \textbf{\textit{Point} – \textit{Slope Form}} \qquad y = y_1 = \boldsymbol{b}$$

Vertical lines cross only the x-axis. Therefore the equation of the line is in the form of x= "some value" and the x-intercept is $(\boldsymbol{x}, \mathbf{0})$.

All other lines must cross both axises. Therefore all other lines with have an x-intercept and y-intercept.

There ***isn't any run (No Run)*** in a vertical line therefore $m = \frac{Rise}{Run} = \frac{rise}{0} = undefined$.

Find the equation of the line in the Slope-Intercept Form using two points.

Example 1: $point\ 1\ (\mathbf{-4,\ 7})$, $point\ 2\ (\mathbf{3,-2})$

$x_1=-4, y_1=7$ $x_2=3, y_2=-2$

1) Use Slope Formula, $m=\frac{y_2-y_1}{x_2-x_1}$. $m=\frac{-2-7}{3-(-4)}=\frac{-9}{7}$ or $\boldsymbol{m=-\frac{9}{7}}$.

2) Choose one of the points (3, -2) and substitute the values of (x, y) with the value of slope into the equation of the line:

$\boldsymbol{y}=\boldsymbol{m}(\boldsymbol{x})+b$ becomes $-2=\frac{-9}{7}(3)+b$ and then solve for ***b***.

$-2=\frac{-9}{7}(3)+b$ a. Simplify

$-2=\frac{-27}{7}+b$

$-2=-3+b$ b. Add **3** to both sides to isolate ***b***.

$\underline{+3\quad +3}$

$1=b$ rewrite as $\boldsymbol{b=1}$

c. Rewrite the equation of the line: $\boldsymbol{y}=\boldsymbol{m}\,x+\boldsymbol{b}$ as $y=\frac{-9}{7}x+1$

Transform the slope-intercept form into the standard form of the equation of the line:

$$\boldsymbol{A}\,x+\boldsymbol{B}\,y=\boldsymbol{C}$$

$$y=-\frac{9}{7}x+1$$

$$\underline{\frac{+9}{7}x \qquad +\frac{9}{7}x}$$

$$\frac{+9}{7}x+y= \qquad 1$$

$$7\cdot\left(\frac{9}{7}x+y\right)=7\cdot(1)$$

$\mathbf{9x+7y=7}$ Standard Form of $\boldsymbol{y=-\frac{9}{7}x+1}$

$$Ax + By = C$$ ***Standard Form***

Finding the x-intercept and y-intercept when the equation is in Standard Form.

a. All line cross the both axises (exception horizontal & vertical lines).

b. Find the values of the **x-intercept** and **y-intercept**.

The **y-intercept** is the point [or coordinate] $(\mathbf{0}, \boldsymbol{y})$ or $(\mathbf{0}, \boldsymbol{b})$, where the line crosses the y-axis. The **x-intercept** is the point [or coordinate] $(\boldsymbol{x}, \mathbf{0})$, where the line crosses the x-axis.

Process
1. Create two columns & label y-intercept (0, y) and x- intercept (x, 0).

y-intercept $(0, y)$ $\quad 6x+3y=36 \quad$ **x-intercept** $(x, 0)$

$6x+3y=36 \qquad\qquad 6x+3y=36$

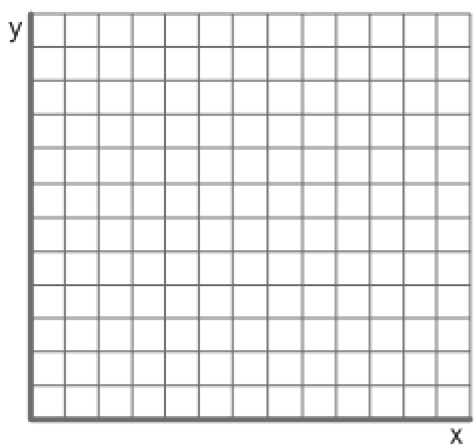
substitute point into equation

$6\cdot 0+\mathbf{3y=36} \qquad\qquad \mathbf{6x}+3\cdot 0=\mathbf{36}$

$\mathbf{3y=36} \qquad\qquad \mathbf{6x=36}$

$\dfrac{3y}{3}=\dfrac{36}{3} \qquad\qquad \dfrac{6x}{6}=\dfrac{36}{6}$

$y=12 \quad (0,12) \qquad\qquad x=6 \quad (6,0)$



Explain: **<u>What I know about lines?</u>**
a. The equation of the line is in standard from.
b. All line cross the both axises (exception horizontal & vertical lines).
c. The y-intercept is (0, y).
d. The x-intercept is (x, 0).

Process
1. Create two columns & label y-intercept (0, y) and x- intercept (x, 0).
2. Write original equation first & then substitute the know value of the appropriate coordinate, y-intercept (0, y) substitute 0 for x in the equation.
3. Simplify equation and solve for y.
4. Write the answer as a coordinate with the value of y. (0, y).
5. Do the same of the x-intercept.
6. Plot the coordinate and draw the line.

The Domain is all values of x
The Range is all values of y.

Parallel Lines $\parallel$ have the same slope and a different y-intercept.

$$\overline{AB} \parallel \overline{CD} \quad m_{AB} = m_{CD}$$

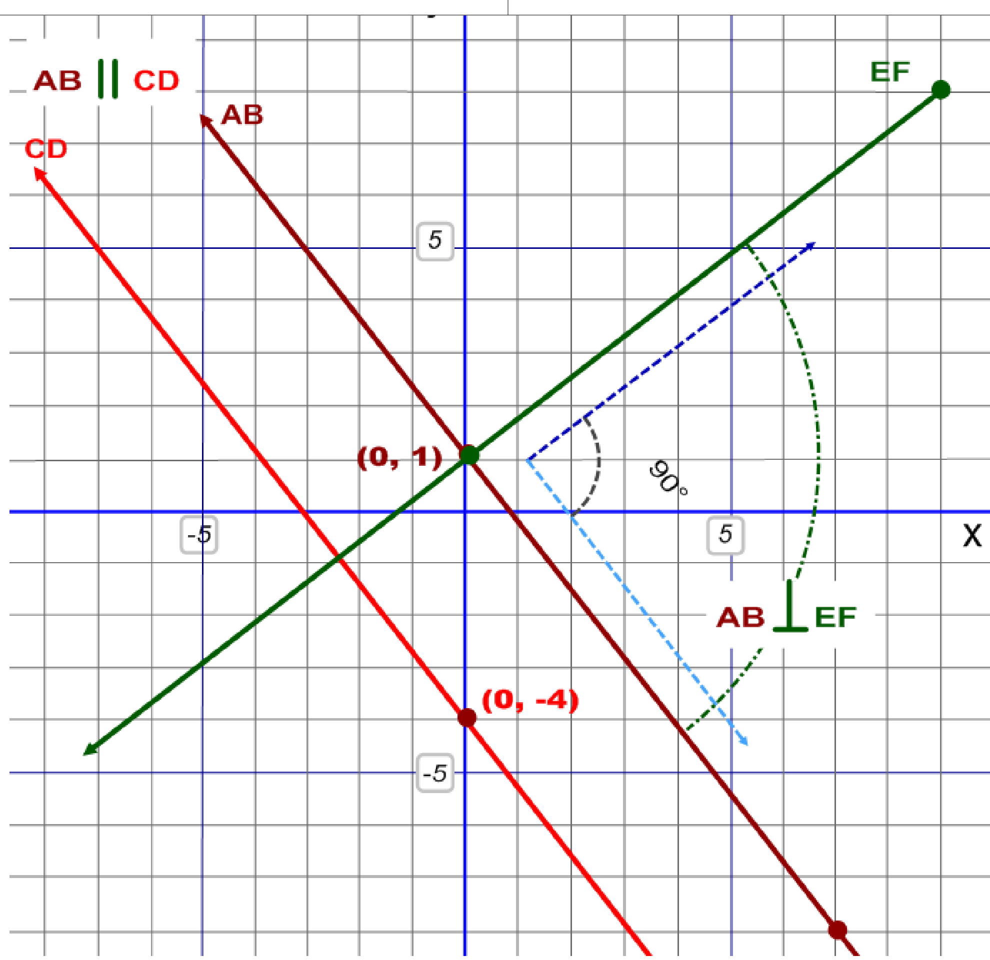
$$\boldsymbol{y = m_{AB}x + b_{AB}} \quad \parallel \quad \boldsymbol{y = m_{CD}x + b_{CD}}$$

Perpendicular Lines $\perp$ $\quad \overline{AB} \perp \overline{EF} \quad m_{EF} = -\frac{1}{m_{AB}}$ or $m_{AB} = -\frac{1}{m_{EF}}$

$$\boldsymbol{y = m_{AB}x + b_{AB}} \quad \perp \quad \boldsymbol{y = m_{EF}x + b_{EF}} \text{ or } \boldsymbol{y = -\frac{1}{m_{AB}}x + b_{EF}}$$

A Perpendicular Line is rotated 90 degrees from the original line.

Parallel Lines have same slope.	**Perpendicular Lines**
$\overline{AB} \quad \boldsymbol{y = -\frac{9}{7}x + 1} \quad m_{AB} = -\frac{9}{7}$	$\overline{AB} \quad \boldsymbol{y = -\frac{9}{7}x + 1} \quad m_{AB} = -\frac{9}{7}$
$\overline{AB} \parallel \overline{CD}$ at $(\mathbf{0, -4})$	$\overline{AB} \perp \overline{EF}$ at $(\mathbf{0, 1})$
$\overline{CD} \quad \boldsymbol{y = -\frac{9}{7}x - 4} \quad m_{CD} = -\frac{9}{7}$	$\overline{EF} \quad \boldsymbol{y = \frac{7}{9}x + 1} \quad m_{EF} = +\frac{7}{9}$



Advanced Parallel & Perpendicular Lines at a Particular point, (x,y) **.**

Find a line that is parallel and perpendicular to $\mathbf{y=-\frac{9}{7}x+1}$ at $(\mathbf{-7,5})$

Use the information above: $m_{AB}=-\frac{9}{7}$ $m_{CD}=+\frac{7}{9}$

Use this the Point-Slope Form of the Equation of the Line.

$y-\mathbf{y_1}=\mathbf{m}(x-\mathbf{x_1})$ $\quad$ ***Point − Slope Form***

Parallel Line at $(\mathbf{-7,5})$	Perpendicular Line at $(\mathbf{9,-5})$
$\overline{AB}$ $\mathbf{y=-\frac{9}{7}x+1}$ $m_{AB}=-\frac{9}{7}$ $y-\mathbf{5}=-\frac{\mathbf{9}}{\mathbf{7}}(x-(\mathbf{-7}))$ $y-\mathbf{5}=-\frac{\mathbf{9}}{\mathbf{7}}(x+\mathbf{7})$ $y-5=-\frac{9}{7}x+(-\frac{9}{7})\cdot 7$ $y-5=-\frac{9}{7}x-9$ $\underline{\;+5 \qquad\qquad +5\;}$ $\mathbf{y=-\frac{9}{7}-4}$ $\overline{AB} \parallel \overline{CD}$ at $(\mathbf{-7,5})$ $\overline{CD}$ $\mathbf{y=-\frac{9}{7}x-4}$ $m_{CD}=-\frac{9}{7}$ y-intercept is $(\mathbf{0,-4})$	$y-(\mathbf{-5})=\frac{\mathbf{7}}{\mathbf{9}}(x-\mathbf{9})$ $y+\mathbf{5}=\frac{\mathbf{7}}{\mathbf{9}}(x-9)$ $y+5=\frac{7}{9}x-(\frac{7}{9})\cdot 9$ $y+5=\frac{7}{9}x-7$ $\underline{\;-5 \qquad\qquad -5\;}$ $\mathbf{y=\frac{7}{9}x-12}$ $\overline{AB} \perp \overline{CD}$ at $(\mathbf{-7,5})$ $\overline{CD}$ $\mathbf{y=\frac{7}{9}x-12}$ $m_{CD}=+\frac{7}{9}$ y-intercept is $(\mathbf{0,-12})$

Constructed Response Type Statements

$y=\boldsymbol{m}x+\boldsymbol{b}$ *Slope– Intercept Form*

Since the equation of the line is in slope-intercept form I know five pieces of information

1. **y-intercept**, $(0,b)$ or $(0,y)$ which is the location where the line passes the y-axis
2. the **slope** of the line ***m***, $m=\frac{Rise}{Run}$
3. the ***rise***
4. the ***run***, and I know
5. if the *slope is positive or negative.*

With this information I can find

1. any number of coordinates from the y-intercept (plot or create a table).
2. the x-intercept, $(x,0)$ which is where the lines passes the x-axis.

I can transform the slope-intercept form $y=\boldsymbol{m}x+\boldsymbol{b}$ into the standard form $\boldsymbol{A}x+\boldsymbol{B}y=\boldsymbol{C}$ and I can transform the standard form ino the slope-intercept using **properties of equality**.

$\boldsymbol{A}x+\boldsymbol{B}y=\boldsymbol{C}$ to $y=\boldsymbol{m}x+\boldsymbol{b}$	$y=\boldsymbol{m}x+\boldsymbol{b}$ to $\boldsymbol{A}x+\boldsymbol{B}y=\boldsymbol{C}$
$\mathbf{3}x-\mathbf{4}y=\mathbf{12}$ $\underline{-3x \qquad -3x}$ $-4y=-3x+12$ $\frac{-4}{-4}y=\frac{-3x+12}{-4}$ $y=\frac{\mathbf{-3}}{\mathbf{-4}}x+\frac{\mathbf{12}}{\mathbf{-4}}$ $y=\frac{\mathbf{3}}{\mathbf{4}}x-\mathbf{3}$ where $\boldsymbol{m}=\frac{3}{4}$ $\boldsymbol{b}=\frac{12}{-4}=-3$	$y=\frac{\mathbf{3}}{\mathbf{4}}x-\mathbf{3}$ $\underline{-\frac{3}{4}x \qquad -\frac{3}{4}x}$ $-\frac{3}{4}x+y=-3$ we could leave it here or modify it to eliminate the fractional coefficient as $(-4)\cdot(-\frac{3}{4}x+y)=-3\cdot(-4)$ multiplied by the **L**owest **C**ommon **D**enominator $\mathbf{3}x-\mathbf{4}y=\mathbf{12}$

We can see the above algebraically.

$Ax+By=C$	$y=mx+b$
$\underline{-Ax \quad -Ax}$ $By=-Ax+C$ $\frac{By}{B}=\frac{-Ax+C}{B}$ $y=\frac{-A}{B}x+\frac{C}{B}$ $y= mx + b$ where $m=\frac{-A}{B}$ $b=\frac{C}{B}$	$\underline{-mx \quad -mx}$ $-mx+y=b$ *If any term is a fraction, multiply all terms by lowest common multiple of denominators to eliminate the fractions.

Transforming from standard form to slope-intercept form is more challenging.

Constructed Response Type Statements II

$Ax+By=C$ ***Standard Form***

- Since the equation of the line is in standard form I can easily find the x-intercept $(x,0)$ and y-intercept $(0,y)$.

The **y-intercept** is the point [or coordinate] $(0,y)$ or $(0,b)$ where a line crosses the y-axis. At this point the x-coordinate is <u>zero</u> and the y-coordinate is <u>b</u> and the unknown value.

The **x-intercept** is the point [or coordinate] $(x,0)$ where a line crosses the x-axis. At this point the y-coordinate is <u>zero</u> and the x-coordinate is the unknown value.

$(0,y)$	$6x+4y=36$	$(x,0)$
$6x+4y=36$ $6\cdot0+4y=36$ $4y=36$ $\frac{4y}{4}=\frac{36}{4}$ $y=9$ therefore $(0,9)$ y-intercept	substitute points into equation	$6x+4y=36$ $6x+4\cdot0=36$ $6x=36$ $\frac{6x}{6}=\frac{36}{6}$ $x=6$ therefore $(6,0)$ x-intercept

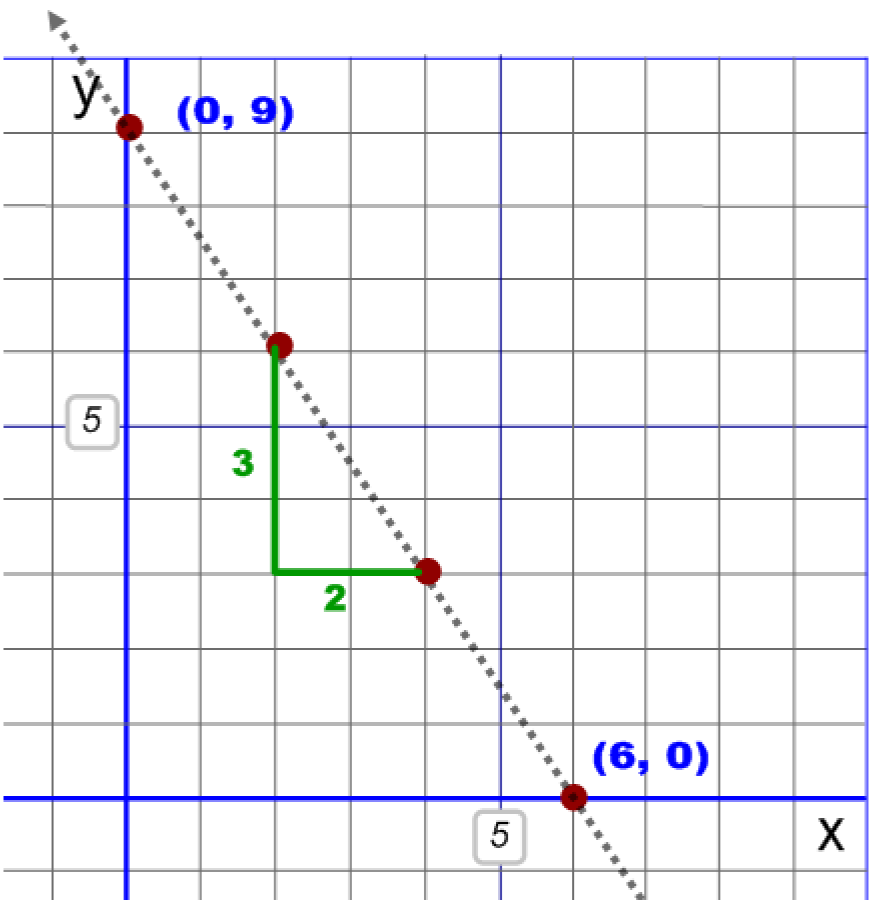
- Point 1 (x_1,y_1) is $(0,9)$.
- Point 2 (x_2,y_2) is $(6,0)$.

- I can find the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ using these two points $(0,12)$ and $(6,0)$.

$$m = \frac{0-9}{6-0} = \frac{-9}{6} = \frac{\mathbf{-3}}{\mathbf{2}} \qquad m = \frac{\boldsymbol{Rise}}{\boldsymbol{Run}} \qquad m = \frac{-3}{2}$$

* Notice the $\frac{\boldsymbol{Rise}}{\boldsymbol{Run}} = \frac{\mathbf{-3}}{\mathbf{2}}$ that is, for every 2 steps to the right we go down by 3.

- Plot on the coordinate grid
- Create a table of the domain and range.



Table

x	y
0	9
2	6
4	3
6	0
domain	*range*